MOST ALL putting greens are neither level nor plane, some being more or less severely contoured and sloped than others. Consequently, Stimpmeter readings, taken over such dissimilar surface profiles, correlate differently as a linear measure of green speed. That is to say, green speed ratings, popularized as they have been by averaging Stimpmeter measurements taken on reasonably level greens, do not fairly and accurately serve as speed indices common to all putting greens. Rather, by preparing an “as built” green to Stimpmeter readings adjusted for its inherent angularities, uniformity of speed can prevail from green to green, stabilizing the composites of golfers and green superintendents in the process.

By mathematically interpreting the physics fundamental to a golf ball rolling over a putting green upon release from a Stimpmeter, indices are derived, as angularity-consistent measures of speed rating characteristic of “as-built” slow-to-fast greens. These indices are graphically plotted to facilitate their use by golf course superintendents, golf committees, tournament officials, and the like.

Modeling Golf Ball Roll

The coefficient of friction between a golf ball and the putting green surface over which it rolls can be quantified by using a Conservation of Energy model as the computational basis for analysis. Stimpmeter measurements, supplemented by green slope measurements over which the Stimpmeter readings are made, are fundamental to the applicability of such an analysis to all putting greens, no matter their angularities or undulations, however severe.

When coefficient of friction values result from Stimpmeter measurements either taken on or normalized to level greens, the measurements range from a low of about 6 feet for what are categorized to be slow greens to a high of about 12 feet for fast greens. But therein lies a rub, because all greens are not level; rather, they are architecturally contoured with slopes, if not marginally, for drainage. Moreover, few putting green slopes are unidirectional; most are compound contoured. Notwithstanding, golf course putting green speeds can be equalized and controlled, over the full range of slow-to-fast, by correlating Stimpmeter and putting green slope readings to coefficient of friction values. Said another way, using Stimpmeter measurements made on level greens as numerical benchmarks to characterize slow-to-fast greens, Stimpmeter readings can be indexed for all 18 golf course greens, having first surveyed their angularities, to comparatively measure up to a desired benchmark speed.

Coefficient of Friction

For the purpose of the analysis, the coefficient of friction can be generalized to encompass, without distinction, the static, dynamic, and rolling coefficients of friction that prevail during the putt of a golf ball starting at rest and rolling to a stop. It can be normalized to an all-inclusive parameter because of its dependence on many variables. Among them, the most influential of which would be the height of cut, are the morphological and growing characteristics of the turfgrass species, the turf density and uniformity, the thatch layer, the dimpling pattern and the construction of the golf ball, the season, the wetness, even the time of day.

Despite the influence of these variables and others, the green speeds of “as built” undulating greens can, with reasonable accuracy, be articulated and prepared analogous to the benchmark green speed indices from Stimpmeter measurements taken on level greens.

Level Putting Surfaces

The mathematical parameters and variables affecting the energy conservation relationships, when making Stimpmeter measurements on a reasonably level putting surface, are depicted in Fig. 1, where:

- $W =$ weight of golf ball, 1.62 oz.
- $H =$ height of Stimpmeter notch above horizontal upon golf ball release, in.
- $O =$ angularity of Stimpmeter notch above horizontal upon golf ball release, 20.5 deg.
- $L =$ Stimpmeter length, 36 in.
- $V_i =$ initial golf ball velocity across the putting surface from the foot of the Stimpmeter, ft./sec.
- $V_f =$ final velocity of golf ball after rolling across the putting surface to a stop, zero
- $S =$ Stimpmeter reading, ft.
- $f =$ coefficient of friction between rolling balls and the putting surface, dimensionless
- $g =$ gravitational acceleration constant, 32.2 ft./sec.

Only $S$ and $f$, as a function of $S$, are variable; the other parameters, in addition to $W, O, L,$ and $g,$ remain constant, to wit:

\[
H = L \sin \Theta \tag{1}
\]

\[
= (36)(\sin 20.5) = (36)(0.350) = 12.6 \text{ in.}
\]

and subsequently, the total Potential Energy, $PE,$ stored in the golf ball prior to release down the Stimpmeter is:

\[
PE = WH = (1.62)(12.6) = 20.4 \text{ in.-oz.} \tag{2}
\]

**Figure 1**
but only a part of which becomes vectorially carried horizontally as an equivalent Kinetic Energy, KE, at velocity, \( V_1 \), or:

\[
KE = PE \cos \theta 
\]

\[
= (20.4)(\cos 20.5) = (20.4)(0.937) = 19.1 \text{ in.-oz.}
\]

the remaining potential energy 20.4 - 19.1 = 1.3 in.-oz. being dissipated as the golf ball impacts vertically to the putting surface from the foot of the Stimpmeter. However:

\[
KE = \frac{1}{2} \frac{1.62}{S} V_1^2 
\]

or

\[19.1 = \frac{1}{2} \frac{1.62}{32.2 \times 12} V_1^2
\]

\[V_1 = \sqrt{\frac{19.1 \times 12}{1.62 \times 12}} = 95.5 \text{ in./sec.}
\]

The Kinetic Energy is all dissipated by frictional resistance as the golf ball rolls to a stop from \( V_1 \) to \( V_0 \) along the Stimpmeter reading. Hence:

\[
KE = W \bar{S} f 
\]

or

\[19.1 = 1.62 \bar{S} f
\]

and transposing the coefficient of friction as a function of the Stimpmeter reading is signified by:

\[
f = \left( \frac{19.1}{12} \right) \frac{1}{\bar{S}} = \frac{0.983}{\bar{S}}
\]

Typical calculated values deriving from Equ(6) are:

<table>
<thead>
<tr>
<th>( \bar{S} ), feet</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.197</td>
</tr>
<tr>
<td>6.0</td>
<td>0.164</td>
</tr>
<tr>
<td>8.5</td>
<td>0.116</td>
</tr>
<tr>
<td>11.0</td>
<td>0.089</td>
</tr>
<tr>
<td>12.5</td>
<td>0.079</td>
</tr>
</tbody>
</table>

These values can be plotted (see Fig. 2) to establish the coefficient of friction, \( f \), for all Stimpmeter readings, \( \bar{S} \), taken on a reasonably level surface.

![Figure 2](image)

Obtaining a realistic Stimpmeter reading on a green that is not level provides challenges to get a number that is representative of the true surface conditions.
Downsloped Putting Surfaces

A similar analysis can be made for a downsloped putting surface, as depicted in Fig. 3. As before, the Potential Energy initially being carried vectorially by the golf ball, as it rolls off from the base of the Stimpmeter, \( \mathbf{W} \cos \theta \). However, as the ball rolls down along the slope to its stopping point, where \( V_x = 0 \), additional PE is acquired by the golf ball the steeper the downslope angle \( \Phi \), the value of which is:

\[
S_{\downarrow} = WX \cos \Phi
\]  

(7)

Accordingly, where \( S_{\downarrow} \) is the Stimpmeter reading taken downhill, the total Potential Energy to be dissipated by friction will be:

\[
WH \cos \theta + WX \cos \Phi = WS_{\downarrow} + f
\]  

(8)

But,

\[
X = S_{\downarrow} \sin \Phi
\]  

(9)

Or,

\[
WH \cos \theta + WS_{\downarrow} \sin \Phi \cos \Phi = WS_{\downarrow} + f
\]  

(10)

And, canceling \( W \) out of each term of Equ. 10 and substituting the value of the fixed parameters, \( H = 12.6 \) inches and \( \cos 20^\circ = 0.937 \),

\[
S_{\downarrow} = \frac{(12.6 \times 0.937)}{f \sin \Phi \cos \Phi} = \frac{11.8}{f \sin \Phi \cos \Phi}
\]  

(11)

From the afore-calculated values of the coefficient of friction, \( f \), over a range of slow to fast Stimpmeter readings taken on a level putting surface, \( S \), the following equivalent values of \( S_{\downarrow} \) can be calculated from Equ. 11, based upon the prevailing downslope angle.

<table>
<thead>
<tr>
<th>( f )</th>
<th>( 0^\circ )</th>
<th>( 1^\circ )</th>
<th>( 2^\circ )</th>
<th>( 3^\circ )</th>
<th>( 4^\circ )</th>
<th>( 5^\circ )</th>
<th>( 6^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.197</td>
<td>5.0</td>
<td>5.48</td>
<td>6.07</td>
<td>6.78</td>
<td>7.74</td>
<td>8.94</td>
<td>10.8</td>
</tr>
<tr>
<td>0.164</td>
<td>6.0</td>
<td>6.71</td>
<td>7.62</td>
<td>8.78</td>
<td>10.5</td>
<td>12.8</td>
<td>16.4</td>
</tr>
<tr>
<td>0.116</td>
<td>8.5</td>
<td>9.98</td>
<td>12.1</td>
<td>15.4</td>
<td>22.1</td>
<td>33.9</td>
<td>81.9</td>
</tr>
<tr>
<td>0.089</td>
<td>11.0</td>
<td>13.8</td>
<td>18.2</td>
<td>26.5</td>
<td>51.8</td>
<td>492</td>
<td>11.09</td>
</tr>
<tr>
<td>0.079</td>
<td>12.5</td>
<td>16.0</td>
<td>22.3</td>
<td>36.4</td>
<td>109</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

Again, these calculated values can be plotted as a family of curves representing the tabulated downslope angles and interpolated in between to establish downhill Stimpmeter readings and, therefore, putting green speeds comparable to such readings and speeds on a level surface. See Fig. 4.

The calculated Stimpmeter reading becomes infinite, \( \infty \), e.g. the ball will not stop rolling downhill, when in Equ. 11 \( \sin \Phi \cos \Phi \) becomes equal to or greater than the coefficient of friction, \( f \). Then

\[
S_{\downarrow} = \frac{11.8}{f \sin \Phi \cos \Phi} = \frac{11.8}{0} = \infty
\]  

(12)

From Equ. 12, the following tabulation can be made of the maximum or limiting downslope angles at which a ball will not stop rolling versus the coefficients of friction that prevail on slow to fast feel putting surfaces.
These values are plotted in Fig. 5 to establish the limiting downslope angles for all values of $S$.

### Upsloped Green Surfaces

Conversely, for golf balls rolling uphill, Stimpmeter reading $S\uparrow$, Eq. 12 becomes

$$S\uparrow = \frac{11.8}{f + \sin \Phi \cos \Phi} \quad (13)$$

Again, the following equivalent values of $S\uparrow$ can be calculated and plotted in Fig. 6 from Eq. 13, based upon the prevailing upslope angle.

<table>
<thead>
<tr>
<th>$f$</th>
<th>$S\downarrow$</th>
<th>Limiting Downslope Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.197</td>
<td>5.0</td>
<td>11° - 40°</td>
</tr>
<tr>
<td>0.164</td>
<td>6.0</td>
<td>9° - 40°</td>
</tr>
<tr>
<td>0.116</td>
<td>8.5</td>
<td>6° - 30°</td>
</tr>
<tr>
<td>0.089</td>
<td>11.0</td>
<td>5° - 10°</td>
</tr>
<tr>
<td>0.079</td>
<td>12.5</td>
<td>4° - 30°</td>
</tr>
</tbody>
</table>

The Brede Formula

By merging Sir Isaac Newton’s motion equations for the up and down slope movements of objects into one equation, A. Douglas Brede developed the following formula (USGA Green Section Record, November/December 1990):

$$\text{Green Speed} = \frac{2 \times S\uparrow \times S\downarrow}{S\uparrow + S\downarrow} \quad (14)$$

### Application

Both the conservation of energy model and the Brede formula have been validated by actual measurements. The conservation of energy model, by including green slope as a variable in calculation, serves to beforehand, having decided upon a desired Stimpmeter speed rating, establish the downhill and/or uphill Stimpmeter readings to which the putting greens need to be prepared. The Brede formula serves, only after the fact, to establish the speed reading that would have prevailed on a level green where, because of green slopes, the traditional two-direction average method would have resulted in an incorrect speed rating.

To measure green slope, a two-man sight-level survey or team need not be used. Instead, one of the new electronic leveling devices, such as a one-man “Smart Level” with a digital readout accurate to ±0.1 degrees serves the purpose. Like the Stimpmeter, it is a simple and fast tool to use.

The sets of curves included in Figs. 4, 5, and 6 define for the golf course superintendent, tournament committees, players, and the like the Stimpmeter readings that should prevail, knowing from prior measurement the angularity of the greens, to control putting speed by indexing these readings to the generally accepted slow-to-fast speed characterizations that result from Stimpmeter measurements made on level greens. For example, at any one golf course, particularly one built with undulating and/or steeply sloped greens, the angularities built into each green can be sectioned and mapped. One of the mapped sections, reasonably consistent in its direction and degree of slope, can then be selected as the basis for Stimpmeter measurement. One or more such sections from the same and other of the putting greens can be similarly selected for Stimpmeter measurement as a check that all are mowed and otherwise groomed consistently in putting speed with each other. As a permanent record, each of the mapped sections can be supplemented with a tabulation of its characteristic green speed versus Stimpmeter readings from Figs. 4, 5, and 6.

Downslope has a much more pronounced effect upon Stimpmeter measurements than does upslope — the steeper the slopes and the faster the cuts, the greater the relative difference. Generalizing, slow-to-medium speed greens, say Stimpmeter reading 5 to 8 on a level green, although they may be undulated upward of 5 to 6 degrees remain reasonably manageable by the golfer. Medium-to-fast greens, say 8 to 12 Stimpmeter reading on a level green, start destabilizing the nerves of golfers when angled upwards of 3 to 4 degrees. Otherwise stated, markedly undulated golf greens, typical of most time-honored courses, would be better maintained with medium-to-slow speed greens, as they had been architecturally conceived to challenge golfers by their contours, not their slickness. To cope with fast greens, surface angularities need be attenuated in fairness to playability by the golfer and, lest we forget, maintenance by the superintendent.

ARTHUR P. WEBER, a chemical engineer by training, has given generously of his time to golf as a USGA Green Section Committee member since 1984. He is a former green chairman of Old Westbury Golf and Country Club, Old Westbury, New York, and past president of the Metropolitan Golf Association.